

## WAVE MEASUREMENT FROM A SUBSURFACE PLATFORM

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**Abstract:** An alternative to the Maximum Likelihood Method (MLM) for directional wave processing is presented for Doppler current profiling type of instruments. The new solution follows a standard triplet analysis for wave directional analysis. The solution uses elements of classic PUV processing as well as the latest in Acoustic Surface Tracking (AST) technology. This new hybrid solution is called the SUV method. One specific advantage is that the SUV solution circumvents the MLM constraint that the Doppler profiler must be static and not moving during the ensemble measurements. The SUV method allows for measurements from a rotating platform such as a subsurface buoy. Results from a directional Waverider (DWR) are compared for a stationary Doppler profiler (AWAC) using the MLM and SUV methods.

### INTRODUCTION

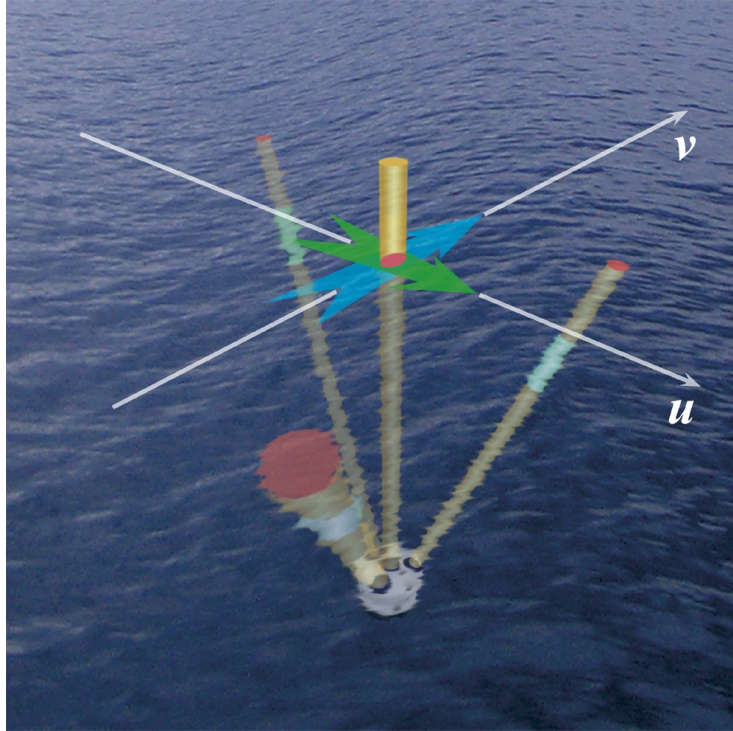
Wave measurements from bottom mounted acoustic Doppler current profilers have circumvented limitations associated with the traditional PUV approach (pressure and horizontal velocity measurements near the instrument) by remotely measuring wave orbital velocities close to the free surface. Here the depth attenuation in the signal is less of a problem, resulting in measurements covering a larger wave frequency range. Thus, acoustic Doppler systems can be mounted at larger depths than the PUV instruments. In addition, the systems are able to measure the average current profile. This effectively provides two measurements from the same instrument.

The Nortek AWAC (Acoustic Wave and Current Profiler) is in this class of Doppler current profilers using the MLM for wave measurements. It performs these measurements using a combination of three slanted acoustic beams, which are symmetrically positioned about the center and angled 25 degrees from the vertical. A vertical fourth beam is dedicated to acoustic surface tracking (AST), which provides direct estimates of the surface elevation.

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**Fig. 1. AWAC pictured below the surface with three current measurement cells and one AST measurement. Beam measurements are transformed to  $U$  and  $V$  for the SUV.**

The AST concept is relatively simple and may be thought of as an inverted echo sounder with a very narrow beam ( $1.7^\circ$ ) which points to the surface. The air-water interface has high acoustic impedance, which leads to a strong surface return. This means that locating the surface and tracking the changes of elevation is a straightforward and robust procedure (Pedersen et al., 2004). The AWAC uses the AST to estimate the frequency spectrum and all non-directional wave parameters.

### **MLM Background**

The MLM methodology used by the AWAC system is a general method for estimating directional wave spectra from spatial arrays of wave measurements (Kahma et al., 2005). Measurements may include surface elevation, surface slope, orbital velocity, and wave induced pressure to name a few. The measurement array for the AWAC is comprised of a center AST measurement and three symmetrical positioned velocity cells (Figure. 1).

Current profilers using the MLM have demonstrated to work well, although they are not without limitations. One clear limitation with the directional part of the spectrum is the size of the array. Spatial aliasing prevents resolving the direction of waves with wavelengths shorter than two times the smallest horizontal separation between two array elements. For the AWAC, these are the AST and one of the velocity measurements. The horizontal separation is depth dependent ( $d \times \sin 25^\circ$ ), leading to an upper frequency limit for wave directions and limits bottom mounted instruments to coastal waters. Thus, an AWAC mounted at 20 meter depth has an upper wave frequency limit of approximately 0.31Hz.

The natural solution to the upper frequency limitation is to mount the AWAC on a submerged buoy held at a constant depth with a vertical mooring line. A submerged platform would allow the AWAC to perform the current profiles and the AST measurements in the traditional way. Unfortunately, assumed rotation of the buoy precludes using the MLM for directional wave processing.

The transfer functions resulting from the spatial lags of the array are essential for the resolution of directional estimates, and a moving instrument will corrupt the algorithm, which assumes a stationary array. In fact, errors in the array geometry are known to affect less robust parameters like the directional spread quite severely. Since a subsurface buoy must be expected to rotate, the position of the velocity measurements will change with time and an alternative technique to MLM must be found for the wave directional estimates.

### **SUV Solution**

One approach to solving this problem is to apply a technique similar to the PUV technique, where we replace the pressure data with the AST data and compute interpolated horizontal velocities  $U$  and  $V$ , vertically aligned with the AST. We shall refer to the method as “SUV”. Estimates of  $U$  and  $V$  are possible since the AWAC is equipped with a compass and tilt sensor which is sampled at the same frequency as the beam velocities. Since the interpolation is carried out instantaneously,  $U$  and  $V$  estimates may be obtained even in the presence of buoy motion.

The transformation from the along beam measurements to  $(U, V)$  applies the standard formula for current profiling instruments. The transformation assumes that currents are uniform within the plane created by the three cells. This assumption is clearly not valid when measuring waves, since the beam cells are spatially separated and therefore the orbital velocities will not be the same at different cells. However, the directional analysis does not need the exact magnitudes of  $U$  and  $V$ . From the definition of the directional Fourier coefficients, it is easily seen that factors multiplying  $U$  and  $V$  will drop out from the definitions of the Fourier coefficients relations as long as the factors are functions only of frequency and equal both for  $U$  and  $V$ . In the present case, the factors have this property to leading order.

The analyses of wave direction can thus be done using simple PUV techniques, where  $P$  is replaced with AST and  $U$ - $V$  are measured close to the surface to accommodate for the attenuation of orbital velocity of short waves.

### **SUV ESTIMATION TECHNIQUE**

The SUV directional estimation procedure is a version of the standard triplet analysis utilizing surface elevation and horizontal velocity in a fixed point. We refer to Kahma et al. (2005) for the derivation of the method, which assumes a directional spectrum of the form  $E(f, \theta) = S(f)D(\theta, f)$ . The directional distribution is written as a Fourier series

$$D(\theta, f) = \frac{1}{\pi} \left[ \frac{1}{2} + \sum_n \{a_n \cos n\theta + b_n \sin n\theta\} \right],$$

and the triplet analysis produces estimates of the first two pairs of Fourier coefficients,

$$a_1(f) = \frac{C_{SU}}{\sqrt{C_{SS}(C_{UU} + C_{VV})}}, \quad b_1(f) = \frac{C_{SV}}{\sqrt{C_{SS}(C_{UU} + C_{VV})}},$$

$$a_2(f) = \frac{\text{Im}[C_{SU}] - \text{Im}[C_{SV}]}{C_{UU} + C_{VV}}, \quad b_2(f) = \frac{2 \text{Re}[C_{UV}]}{C_{UU} + C_{VV}}.$$

where  $C_{**}$  are the cross spectra indicated by the indices.

Standard directional parameters are the frequency dependent mean wave direction,  $\theta_1(f) = \arctan 2(b_1(f), a_1(f))$ , and directional spreading,  $\sigma(f) = [2(1 - r_1(f))]^{1/2}$ ,  $r_1 = \sqrt{a_1^2 + b_1^2}$ . The parameters may be averaged over various frequency bands, or calculated at the peak frequency ( $f_p$ ) of the energy spectrum, as given by the AST power spectrum. Hence, the peak wave direction is  $\theta_{peak}(f) = \arctan 2(b_1(f_p), a_1(f_p))$ .

## EXPERIMENTAL RESULTS

As an initial evaluation of the performance of the SUV approach, we have used the data from a bottom mounted AWAC. This provided a direct comparison between the SUV method and the MLM using the same sensor data. The test was conducted at Diablo Canyon, California (Figure 2). Diablo Canyon is located approximately 150 nautical miles northwest of Los Angeles, California. The location was chosen because of an existing DWR buoy, which provides comparative results. Here a 1 MHz AWAC was mounted on the sea floor at a 25 meter depth and in close proximity (20 meter horizontal separation) to the DWR.

The DWR buoy is part of CDIP (Coastal Data Information Program), which has several buoys along the California coast. CDIP makes the data from these buoys available at their web site (<http://cdip.ucsd.edu>). The fact that the DWR data are regularly posted on the website meant that there was an easy means of retrieving and evaluating the DWR data for this study.

### Deployment Setup

The comparison test was conducted for a period of 23 days. However, only the first 7 days we useable for a rigorous comparison due to an unfortunate loss of the Waverider, which apparently broke free from its mooring. A replacement was in place 8 days later, but as the comparison shows, this DWR was possibly not functioning properly and most likely required calibration. For this reason, the subsequent comparison focuses only on the first DWR.



**Fig. 2. Diablo Canyon, California test site for the AWAC and Datawell DWR.**

The CDIP's DWRs are configured to collect approximately 27 minutes of data at 1.28 Hz, performed twice an hour. The AWAC was set up to profile current velocities in 1 m cells every 20 minutes; while waves were measured once an hour at 1 Hz (2 Hz for the AST) for 34 minutes. The 23 days of data resulted in 546 wave ensembles. The two instruments were slightly out of sync; the AWAC began 1 minute after the DWR. This meant that 25.5 minutes of the respective ensembles overlapped.

### Processing

The AWAC has three independent methods of measuring and estimating the non-directional wave spectra. These are based on the pressure signal, the near surface velocity measurements, and the Acoustic Surface Tracking (AST). The advantage of having three independent estimates for the non-directional spectra is that we are able to use this as an internal check to verify that the system as a whole is functioning properly.

The pressure-based estimates follow the standard linear wave theory transformations to arrive at the surface spectra. It therefore is capable of only measuring the longer waves (approximately 6 seconds and longer for a 25 meter deployment depth).

Much of the AWAC's success can be attributed to the AST. This measurement does not suffer from the depth limitations like the other two methods. Furthermore, it provides a direct measurement of the free surface, as opposed to inferred estimates from either the velocity or the pressure. The AST is also included in the MLM solution, which provides better directional estimates than just the purely velocity based solution.

The post processing of the AWAC data was done in such a way that most closely matched the processing of the DWR. The DWR performs an FFT on 256 samples (200 seconds at 1.28 Hz). The average of 8 of these spectra is reported once every half hour to provide 16 degrees of freedom on 1600 seconds of data.

The difference in the sampling scheme for the AWAC (2048 samples at 1 Hz) and spectral smoothing approach (band averaging in frequency domain) meant that the same exact smoothing could not be employed. However, a similar band average was achieved using 20 degrees of freedom in the frequency domain. Wave data was processed for the band between 0.025-0.25 Hz for directional estimates and 0.025-0.5 Hz for non-directional estimates.

### **Analysis**

We first present the data with the bulk wave parameters significant wave height, mean period, and mean direction. These integral parameters are common for understanding and describing waves although, when comparing wave measurement systems, it is sometimes more helpful to evaluate similar estimates on a band-by-band basis. The subsequent band analysis compares band averaged estimates of energy, direction, and spreading.

### **Bulk estimates**

These estimates are an efficient manner to provide general picture into the way we understand the full wave distribution. The most familiar estimate, significant wave height, indicates the total energy by means of a meaningful value (height). The mean period provides an indication of energy distribution in frequency space. The mean direction provides and indication of the primary wave direction and it is an energy weighted direction estimate. Errors with the distribution of energy will clearly have a negative influence on these estimates. The estimate definitions are:

$H_s = 4\sqrt{M_0}$ ,  $T_{mean} = \sqrt{M_0/M_2}$ , where the moments  $M_n$  are defined as

$$M_n = \int C_{SS} f^n df, C_{SS} \text{ is the energy spectra from the AST.}$$

The directional estimates can be described in terms of the mean direction at each frequency or a average direction, which is an energy weighted estimate over a frequency band. These estimates are respectively,  $\theta_{mean} = \arctan 2(a_1, b_1)$ ,  $\theta_a = \arctan 2(a_a, b_a)$ . The mean direction is based on the first pair of Fourier coefficients at each frequency, whereas the average direction is defined by energy weighted estimates;

$$a_a = \frac{\int C_{SS}(f) \cos(\theta_{mean}(f)) df}{\int C_{SS}(f) df},$$

$$b_a = \frac{\int C_{SS}(f) \sin(\theta_{mean}(f)) df}{\int C_{SS}(f) df}.$$

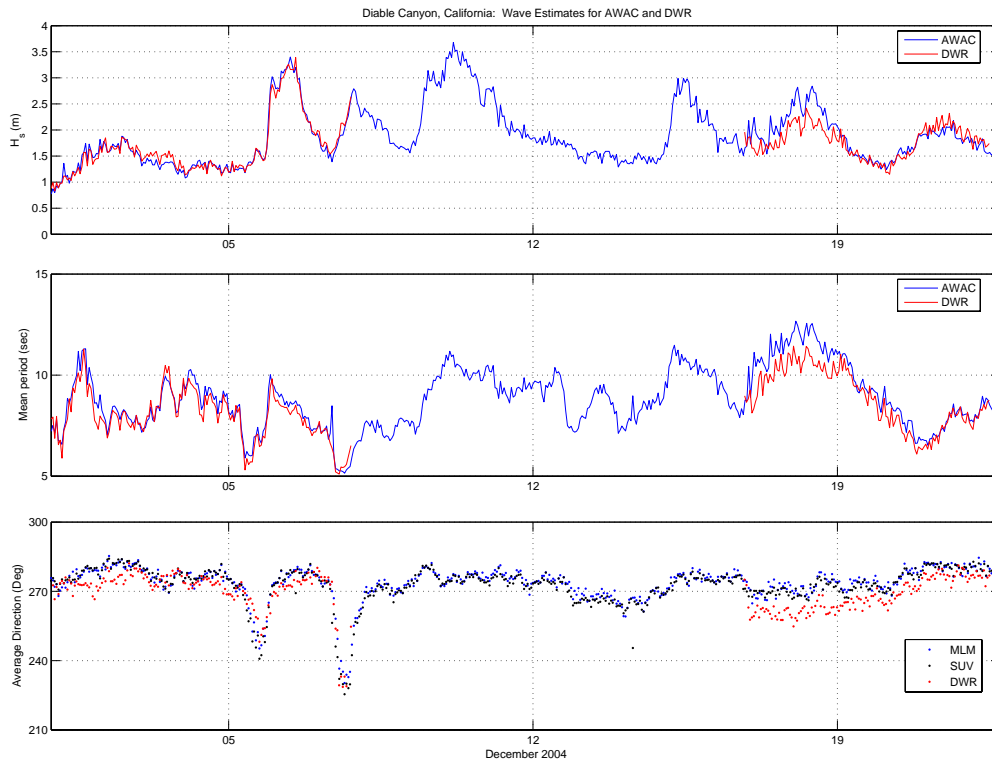
Figure 3 shows the time history plot of these estimates. The plots show the full three weeks of data collection of the AWAC with both the original DWR during the first week and the replacement DWR during the last week. The most meaningful comparison is the first 7 days with the original DWR. As we can see, the estimates for  $H_s$  have very good agreement. The mean period estimates again indicate good agreement, but the DWR tends to estimate lower mean period. The mean direction estimates of both instruments are similar however, there appears to be a mean difference between the two; the DWR tends to estimate lower direction (1.8 degree bias). The overall trends for all estimates agree well.

### **Band Analysis**

CDIP commonly provides energy and directional estimates for nine separate bands. For brevity, we perform a similar analysis over three bands: 14-25, 8-14, and 4-8 seconds. The hope is to separate waves that have long, intermediate, and short wavelengths, and characterize performance differences between the directional estimation methods.

As indicated previously, the second DWR was possibly deployed without proper calibration. Therefore, energy and directional estimates from the period of December 17 – 22, should be evaluated with caution. The initial indication of the comparison is that all estimators of the energy and direction are very similar. We note that the energy estimates have better agreement between the AWAC's AST and pressure measurements, than either has with the DWR. This may be attributed to the time difference for sampling. The poor agreement of the energy estimates from the pressure in the band of 4-8 seconds is attributed to information loss of the highly attenuated short waves below 6 seconds.

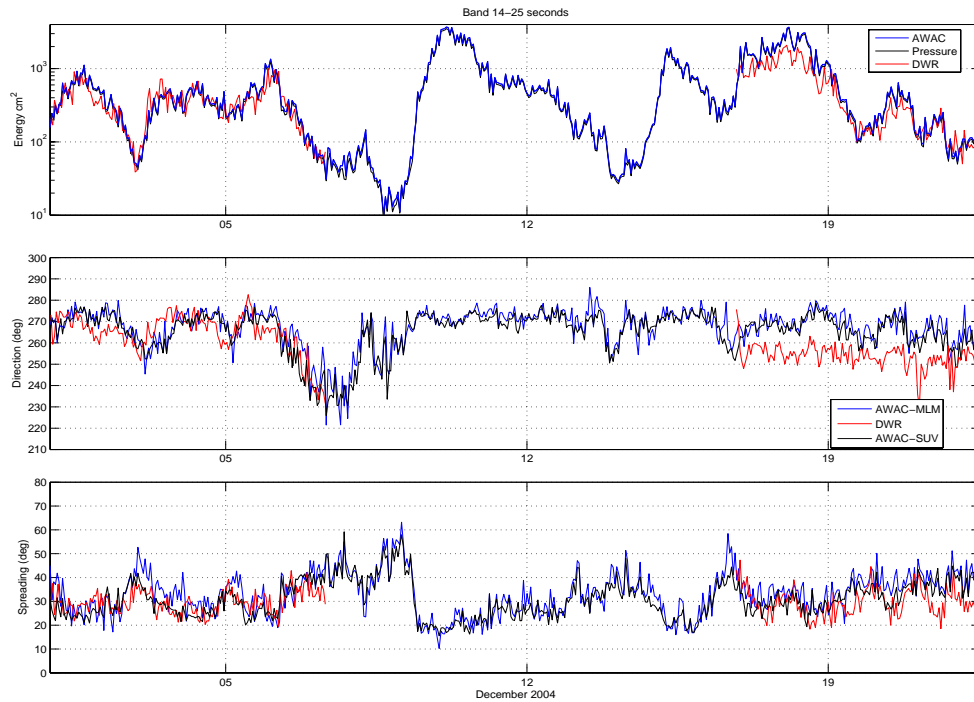
The wave directional estimates for the three estimators are also very close in agreement. Again, there seems to be greater difference between the AWAC's two estimators and the DWR; however, the difference tends to be no more than 10 degrees.



**Fig. 3. (a)  $H_s$ , (b) Mean Period, (c)  $\theta_{ave}$ . AWAC-blue, DWR-red, SUV-black,.**

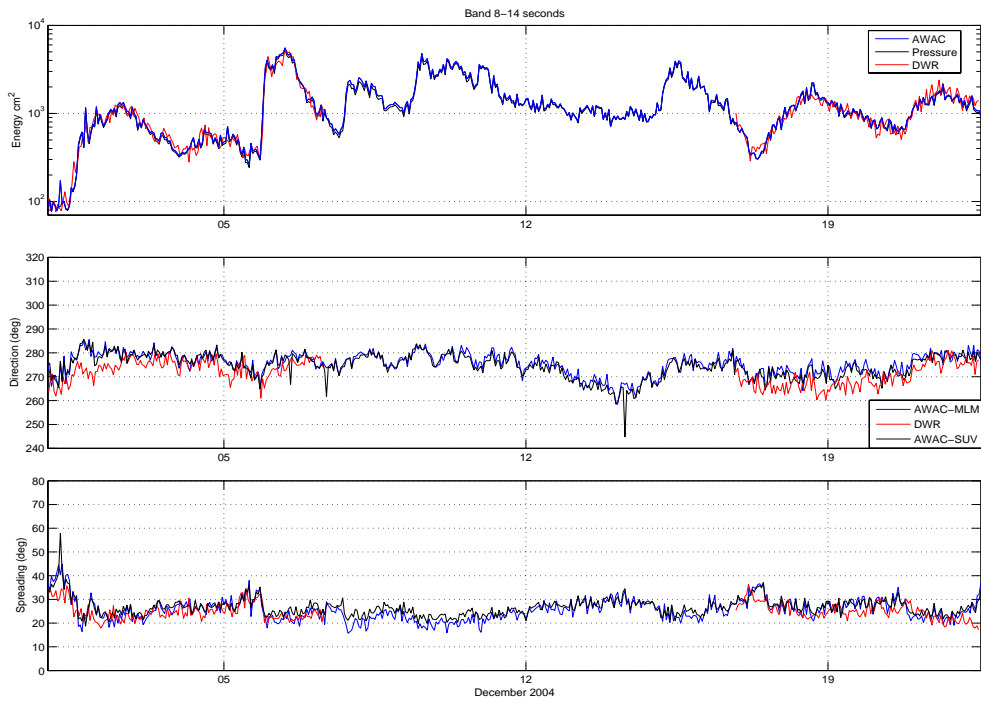
There also seems to be a slight bias in the difference, this is most evident in the intermediate band of 8-14 seconds. The SUV and MLM mean direction estimates again seem to be again in better agreement and this is most likely attributed to the like time series. It is encouraging to see that the two different directional solutions provide very agreeable results. The band associated with the shortest waves show very good agreement as the direction changes over 90 degrees in a relatively short time period.

The directional spread estimates show the greatest difference. It is interesting to see that the SUV solution provides lower directional spread estimates than the MLM for the lowest band, and more closely agrees with the DWR's directional spread estimate. The intermediate band shows the best agreement and may be attributed to the bands high energy level. The short wave band shows the greatest disparity, where the DWR estimates lower spreading when the wave energy is less than  $400 \text{ cm}^2$ .

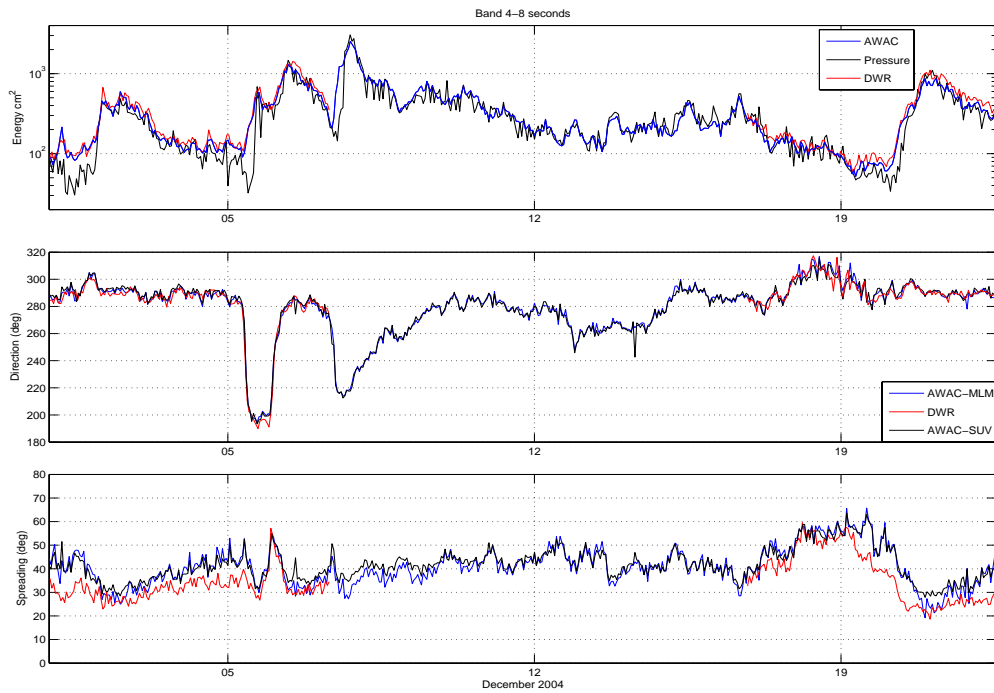


**Fig. 4. Low frequency band: 14-25 seconds, (a) Energy, (b) Average direction, (c) Spread. Red is DWR, blue is AWAC-MLM, black is AWAC-SUV.**





**Fig. 5. Low frequency band: 8-14 seconds, (a) Energy, (b) Average direction, (c) Spread. Red is DWR, blue is AWAC-MLM, black is AWAC-SUV.**



**Fig. 6. Low frequency band: 4-8 seconds, (a) Energy, (b) Average direction, (c) Spread. Red is DWR, blue is AWAC-MLM, black is AWAC-SUV**

## CONCLUSIONS

A new approach to processing wave data from a combined current profiler and Acoustic Surface Tracking instrument is presented. The solution offers the distinct advantage that it may be mounted on a moving platform such as a subsurface buoy. The SUV solution circumvents the complications experienced with the traditional MLM, for which the solution becomes prohibitively complicated for measurements from a moving platform.

The SUV applies special treatment of the orbital wave current measurements to allow for a triplet type of solution. Comparative data collected at Diablo Canyon, California for an AWAC and Directional Waverider show that the SUV solution for a bottom mounted AWAC compares favorably with the DWR as well as the standard MLM solution.

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